

MEMORANDUM

RM-3189-PR

SEPTEMBER 1962

A METHOD OF COMPUTING
THE DISTRIBUTION FUNCTION OF
RADIOACTIVE DOSES FROM MULTIPLE
NUCLEAR DETONATIONS

E. S. Batten

PREPARED FOR:

UNITED STATES AIR FORCE PROJECT RAND

The **RAND** *Corporation*
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum, a further development of techniques described in RM-2734, substitutes for earlier graphical methods the Monte Carlo machine method, thereby allowing extension to the more complicated problem of multiple targets while taking into account the effects of wind variability. It is part of a continuing Project RAND study of radioactive fallout.

SUMMARY

Present methods of predicting fallout can estimate the contamination resulting from a multibomb campaign in an extensive area by applying the random bomb-drop technique or by computing the fallout for a given set of targets and wind patterns. For multiple targets near population centers, we need a new computational method that can be applied to a small geographical area and that will account for the overlapping of fallout patterns encountered in a variety of wind conditions. This Memorandum presents a possible method, develops it, and applies it to a typical target system. The final section discusses the limitations imposed by uncertainties in the initial assumptions.

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I. INTRODUCTION

The potential hazard of radioactive fallout in nuclear war has been discussed at great lengths. To measure its effects on the population, both simple and complex fallout models have been developed. But to date, none of these models can reliably estimate the hazard. They are all affected by uncertainties in the original information — especially those concerning the meteorological conditions during and after the explosion. This report presents a method of estimating fallout that accounts for wind variability in a small geographical region when the targets are known and the weight of the attack is parameterized.

One previous method has been that of the random bomb drop.⁽¹⁾ It postulates an attack in which bombs of equal yield are dropped at random in an area that is large relative to the area contaminated by a single bomb. It assumes that areas contaminated by at least a certain level of radiation are less sensitive to prevailing wind conditions than to the weapon yield. Over a large range of doses, yields and wind conditions, the area contaminated by at least a given level can be considered directly proportional to the yield.⁽²⁾ Since the wind over a large area is extremely varied, we assume that the randomness of bomb drops eliminates any wind-induced deviations from the dose-area relationship. Therefore, we consider the dose-area relationship for a given yield to be constant for all wind conditions. Using these assumptions in a statistical model, we can estimate the expected fraction of the total area that would be covered by more

than a given dose. The method cannot determine, however, the geographical distribution of the dose. If we must know the location of areas contaminated by various doses, we must know the location of the detonations.

To estimate the hazard of an attack on a given system of targets over an extensive area, we need another method. Here the wind variation cannot be eliminated so easily. Wind is a complex function of the three space coordinates and time. On a given day over an area as large as the United States, the horizontal wind flows through an intricate wave pattern in the pressure field. To know the fallout distribution resulting from an attack on given targets in such a large area, we would have to consider the spatial variation of the wind. Determining the probability of this pattern would magnify the problem. We could solve it, in theory, if we had a detailed statistical description of the wind field in space and time. Without such statistics, computations from a judicious choice of wind regimes would be valuable.

We now have two methods. The first emphasizes the fraction of the total area covered by a given dose or greater. For example, it may tell us that $1/25$ of the total area of the United States is covered by greater than 1000 roentgens, but not whether this includes Bangor, Little Rock, or Azusa. The second method emphasizes the location of areas exposed to various doses for a given set of targets and for several given wind patterns over the area. But no attempt is made to determine the probability that these wind patterns will indeed occur. In both methods, we estimated the contamination resulting from a multibomb campaign in an extensive area. Although both methods can provide

valuable estimates of the fallout hazard to the nation as a whole, they cannot provide a detailed analysis of the effects of wind variability on specific areas.

For this reason, we developed a third method — one that would estimate the effects of wind variability on fallout in the immediate environs of a single detonation or of a set of closely spaced detonations. That this method is not restricted to trivial problems is evident from the existence of multiple targets in the vicinity of population centers. With such multiple targets, one wants to know both the horizontal distribution of expected doses and the distribution function of doses for the area surrounding the detonations.

Sections II, III, and IV will discuss the fallout model and wind statistics, then combine them in a statistical model that is designed to compute these distributions. We will apply this statistical model to a typical target system and present the results in Section V. The final section contains a general discussion of the importance of other parameters in the model, such as weapon characteristics and targeting.

II. FALLOUT MODEL

Within minutes of a nuclear detonation, a cloud of radioactive particles of various sizes is formed in the atmosphere. The role of the fallout model is to predict the distribution of radioactivity on the ground. The fallout model we will use in this Memorandum was discussed in detail elsewhere.⁽³⁾ In brief, the model assumes:

1. The radioactive cloud is a right circular cylinder whose height and dimensions are functions of the yield.

$$\begin{aligned}
 R_c &= 8.06 W^{0.45} && \text{(Nautical miles)} \\
 H_B &= 26.8 W^{0.1} && \text{(Kilofeet)} \\
 \Delta H &= 22.0 W^{0.1} && \text{(Kilofeet)} \\
 \mathcal{V} &= 4500 W && \text{(n mi}^2 \text{ kilofeet)}
 \end{aligned}$$

In these and in later equations, R_c is the radius of the cloud, W is the yield, H_B is the height of the base, ΔH is the thickness of the cloud, \mathcal{V} is the volume of the cloud, A is the activity, r is the radius of the particle, H is a certain height in the cloud, and R is the radial distance.

2. The radioactivity is distributed with particle size according to a log-normal law.

3. The radioactivity is uniformly distributed within the cloud. Thus if the initial distribution of activity in the cloud — $A(r, H, R)$ — is assumed to be the product of three independent distributions, we have

$$A(r, H, R) = A_1(H)A_2(R) \frac{1}{\sqrt{2\pi} \sigma} \frac{e^{-1/2 \left(\frac{\ln r - \mu}{\sigma}\right)^2}}{r} \quad (1)$$

But $A_1(H)$ and $A_2(R)$ are constant within the cloud by assuming uniformly distributed activity. Furthermore, because the integral of (1) over the volume of the cloud and over every particle size must be 1, we find that the product $A_1(H)A_2(R)$ equals the reciprocal of the cloud's volume.

Thus,

$$A(r,H,R) = \frac{1}{\gamma} \frac{1}{\sqrt{2\pi}\sigma} \frac{e^{-1/2(\ln r - \mu)^2}}{r} \quad (2)$$

4. A particle of radius r , initially at a height H , will fall to the ground in the time T given by

$$T(r,H) = \alpha + \frac{\beta H}{r^{1.5}} \quad (3)$$

5. The wind affecting the fallout particles can be approximated by a vertically averaged wind vector.

To determine the fraction of the total radioactivity arriving at an area X n mi downwind and Y n mi crosswind, Eq. (2) must be integrated over every particle size arriving at the point (X, Y) . Thus by integrating over every particle size falling from a height H and then over every height from the bottom to the top of the cloud, the fraction of the total radioactivity becomes

$$F(X,Y) = \frac{1}{\gamma \sqrt{2\pi} \sigma} \int_{H_B}^{H_T} \int_{r_1(H)}^{r_2(H)} e^{-1/2(\frac{\ln r - \mu}{\sigma})^2} d(\ln r) dH \quad (4)$$

where $F(X,Y)$ is the fraction of the total radioactivity at (X,Y) , $r_1(H)$ and $r_2(H)$ are the particle sizes at H coming from the front

and rear edges of the cloud, and H_B and H_T are the heights of the cloud's base and top.

A more convenient form can be obtained by using Eq. (3) to transform Eq. (4) into a time-particle size integration. Thus, $F(X,Y)$ may be written as

$$F(X,Y) = \frac{1}{q} \int_{T_1}^{T_2} \int_{r_1(T)}^{r_2(T)} \frac{r^{0.5}}{\beta \sqrt{2\pi} \sigma} e^{-1/2 \left(\frac{\ln r - \mu}{\sigma}\right)^2} dr dT \quad (5)$$

where $r_1(T)$ and $r_2(T)$ are the smallest and largest particles at (X,Y) arriving at time T , and T_1 and T_2 are the earliest and latest times of arrival. If we now define

$$\psi'(T) \equiv \int_{r_1(T)}^{r_2(T)} \frac{r^{0.5}}{\beta \sqrt{2\pi} \sigma} e^{-1/2 \left(\frac{\ln r - \mu}{\sigma}\right)^2} dr \quad (6)$$

then

$$F(X,Y) = \frac{1}{q} \int_{T_1}^{T_2} \psi'(T) dT$$

which may be approximated by

$$F(X,Y) \approx \frac{\Delta T}{q} \psi'(\bar{T}) \quad (7)$$

At (X,Y) , the fallout's duration is ΔT (the difference between T_2 and T_1), while its mean arrival time is \bar{T} (the average of T_2 and T_1). Values of $\psi'(\bar{T})$ computed from Eq. (6) for a one megaton

weapon are shown in Fig. 1.* The function $\psi'(\bar{T})$ in Fig. 1 may be scaled for other yields by using the equation

$$\bar{T}^* = \alpha + \frac{\bar{T}_W - \alpha}{W^{0.1}} \quad (8)$$

For a given mean time of arrival (\bar{T}_W) of fallout from a weapon of yield W , Eq. (8) computes a hypothetical time (\bar{T}^*) so that $\psi'(\bar{T}_W)$ will be identical to $\psi'(\bar{T}^*)$, the value of the ψ' function for a one megaton weapon at the hypothetical time \bar{T}^* .

Both ΔT and \bar{T} are functions of T_2 and T_1 , themselves functions of the wind velocity. Nevertheless, T_1 (the earliest time of arrival) can never be less than 0.23 hr. This is the approximate time of fall for the heaviest particle and approximately the earliest fallout arrival time observed in Bikini tests.⁽⁴⁾ Thus for areas at distances less than $0.23\bar{v}$ nautical miles downwind from the leading edge of the cloud, T_1 will always be 0.23 hr. The last particles will fall from the trailing edge of the cloud in time T_2 given by

$$T_2 = \frac{X + R_c \sqrt{1 - (Y/R_c)^2}}{\bar{v}}$$

* Subsequent to the release of Ref. 3, we decided that the ψ' functions in that report would not provide enough fallout near ground zero. To remedy this, the lower one-fifth of the cloud was assumed to contain a second distribution of activity with particle size. The second distribution contains larger particles and accounts for five per cent of the total radioactivity. Figure 1 is the sum, therefore, of two properly weighted ψ' functions.

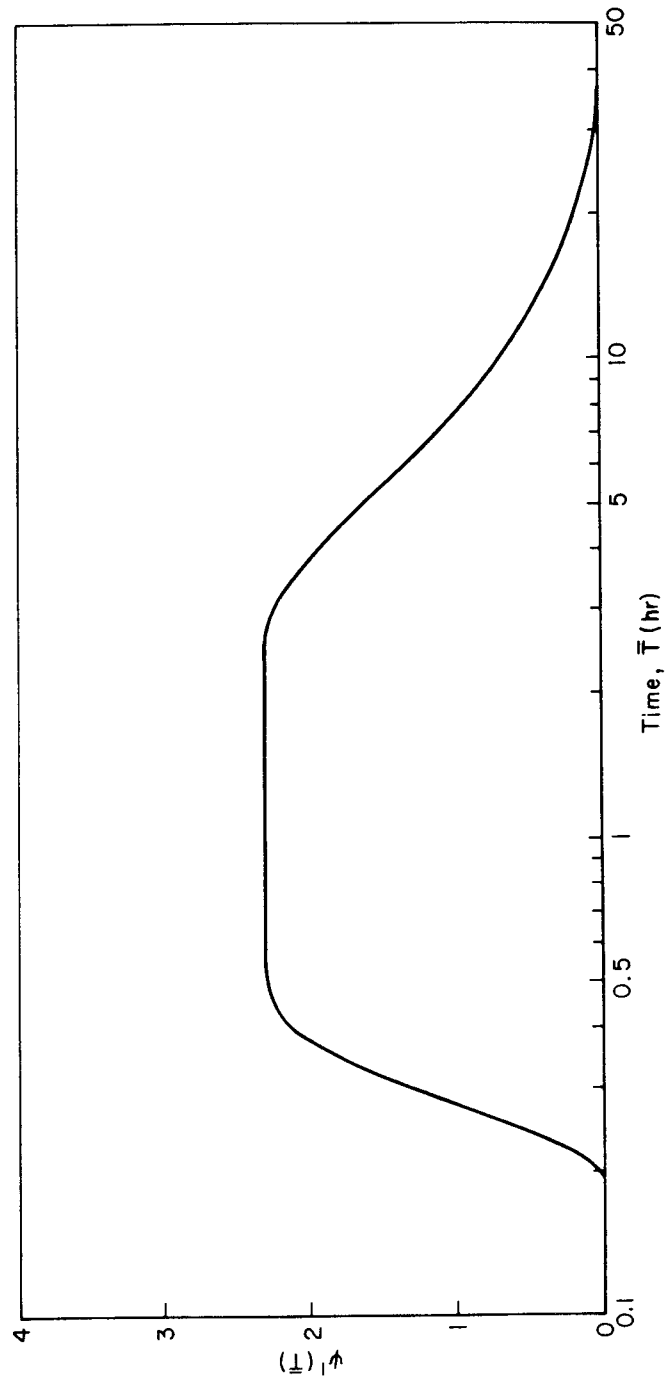


Fig. 1—The function $\psi'(\bar{T})$ vs time for IMT weapons

where \bar{V} is the mean wind velocity. Therefore, when the downwind distance X is less than $R_c \sqrt{1 - (Y/R_c)^2} + 0.23 \bar{V}$, the mean arrival time and the duration of the fallout at (X, Y) become:

$$\bar{T} = \frac{T_2 + T_1}{2} = \frac{X + R_c f(Y) + 0.23 \bar{V}}{2\bar{V}} \quad (9a)$$

$$\Delta T = T_2 - T_1 = \frac{X + R_c f(Y)}{\bar{V}} - 0.23 \quad (9b)$$

where $f(Y) = R_c \sqrt{1 - (Y/R_c)^2}$. When X is greater than $R_c f(Y) + 0.23\bar{V}$ the earliest time of arrival is

$$T_1 = \frac{X - R_c f(Y)}{\bar{V}}$$

and therefore the equations become

$$\bar{T} = \frac{X}{\bar{V}} \quad (10a)$$

$$\Delta T = \frac{2R_c f(Y)}{\bar{V}} \quad (10b)$$

The equations discussed so far will provide $F(X,Y)$, the fraction of the total fissionable material in an area of 1 n mi^2 around the point (X,Y) . To convert this fraction to a measure of the radioactivity in the contaminated area, $F(X,Y)$ must be multiplied by (a) the fission yield of the weapon, fw , where f is the fraction of the yield due to fission and w is the yield, (b) the number of gamma megacuries produced by one megaton of fission at one hour, and (c) the number of roentgens per hour produced by one megacurie of gamma radiators spread



over 1 n mi^2 . Thus, the one-hour hypothetical dose rate* is

$$R_1' = M f w F(X,Y) \quad (11)$$

Here M is the product of the factors in (b) and (c) above, and represents the dose rate at one hour after detonation, per megaton of fission per square nautical mile. Estimates of the value of M range from 9×10^5 to 20×10^5 r/hr/MT/(n mi)². Assuming that the radioactivity decays as $t^{-1.2}$, the dose rate at any time, T, becomes

$$R'(T) = R_1' t^{-1.2} \quad (12)$$

and the integrated dose received T hours after the detonation is

$$R(T) = R_1' \int_{t_a}^T t^{-1.2} dt = R_1' \left[(t_a)^{-0.2} - (T)^{-0.2} \right] \quad (13)$$

where t_a is the time of the fallout's arrival. The value of t_a is approximated by \bar{T} in Eqs. (9) and (10).

Thus Eq. (5), with the appropriate values of ΔT and \bar{T} from Eqs. (9-10), is used to compute $F(X,Y)$. This fraction is then converted to the hypothetical one-hour dose rate at (X,Y) by using Eq. (11). The degree of exposure to radioactivity can then be expressed in terms of either the dose rate at time T by Eq. (12) or the integrated dose by Eq. (13).

* The dose rate is defined as the amount of nuclear radiation to which an individual would be exposed per unit time. The hypothetical one-hour dose rate is the value one hour after the explosion if the fallout had then been complete.

III. WIND STATISTICS

The fallout model discussed in the preceding section assumes that the wind can be approximated by a vertically averaged wind vector. This is a gross simplification of the wind structure affecting the fallout particles. We might ask whether or not such a simple representation of the wind is sufficient for fallout predictions. The vertically averaged wind is obviously insufficient in problems requiring a fallout pattern from a single detonation on a given day. Directional wind shear (when the wind direction changes with height) and — to a lesser extent — speed shear could produce a fallout pattern vastly different from one predicted by using a single wind vector. If there is no directional shear, however, a vertically averaged wind vector is adequate. The fallout distribution will then appear as a cigar-shaped pattern symmetrical with an axis drawn along the mean wind direction. With directional shear, the fallout would extend downwind in the mean-wind direction, it would increase laterally, appearing asymmetric to an axis along the mean wind.

With or without shear, bombs of equal yield and fission fraction will produce the same amount of fallout. But with shear, the fallout will spread over a greater area, thereby decreasing the maximum level observed at a given distance from ground zero. Here we want to add wind statistics to the fallout model by essentially integrating over every possible wind speed and direction. Sometimes by neglecting directional shear and therefore the lateral spreading, we will overestimate the dose in areas near the center of the pattern. Sometimes,

in contrast, because of the pattern's smaller width we will underestimate the dose in areas near the edge of the pattern. Since the two effects act in opposition, any errors produced by neglecting shear should be small. A statistical distribution of the vertically averaged wind should be sufficient.

To provide the necessary wind statistics, the true distribution of the wind was approximated by a circular normal distribution. For the assumed circular normal distribution, the mean and standard deviation of the wind can be approximated by:

$$\bar{V} = \sum_{i=1}^n \rho_i V_i$$

$$S = \sqrt{\frac{S_x^2 + S_y^2}{2}}$$

where

$$S_x^2 = \sum_{i=1}^n \sum_{j=1}^n \rho_i S_{xi} \rho_j S_{xj} R_{ij}$$

$$S_y^2 = \sum_{i=1}^n \sum_{j=1}^n \rho_i S_{yi} \rho_j S_{yj} R_{ij}$$

Here, ρ_i and ρ_j are the weighting factors at the altitudes i and j , R_{ij} is the correlation coefficient between the winds at the altitudes i and j , and S_{xi} , S_{xj} , S_{yi} and S_{yj} are the standard deviations of the x (east-west) and y (north-south) component of the wind at the levels i and j . The weighting factor ρ_i (or ρ_j) is the fraction of the time that the particles spend in the atmospheric layer bounded by the altitudes $i(j)$ and $i + 1 (j + 1)$.

Circular normal distributions of the fallout wind for the summer and winter season were computed for 17 locations in the United States and Canada. Maps of the mean east-west wind (\bar{u}), the mean north-south wind (\bar{v}), and the standard deviation of these components (S) are given in Figs. 2-7.

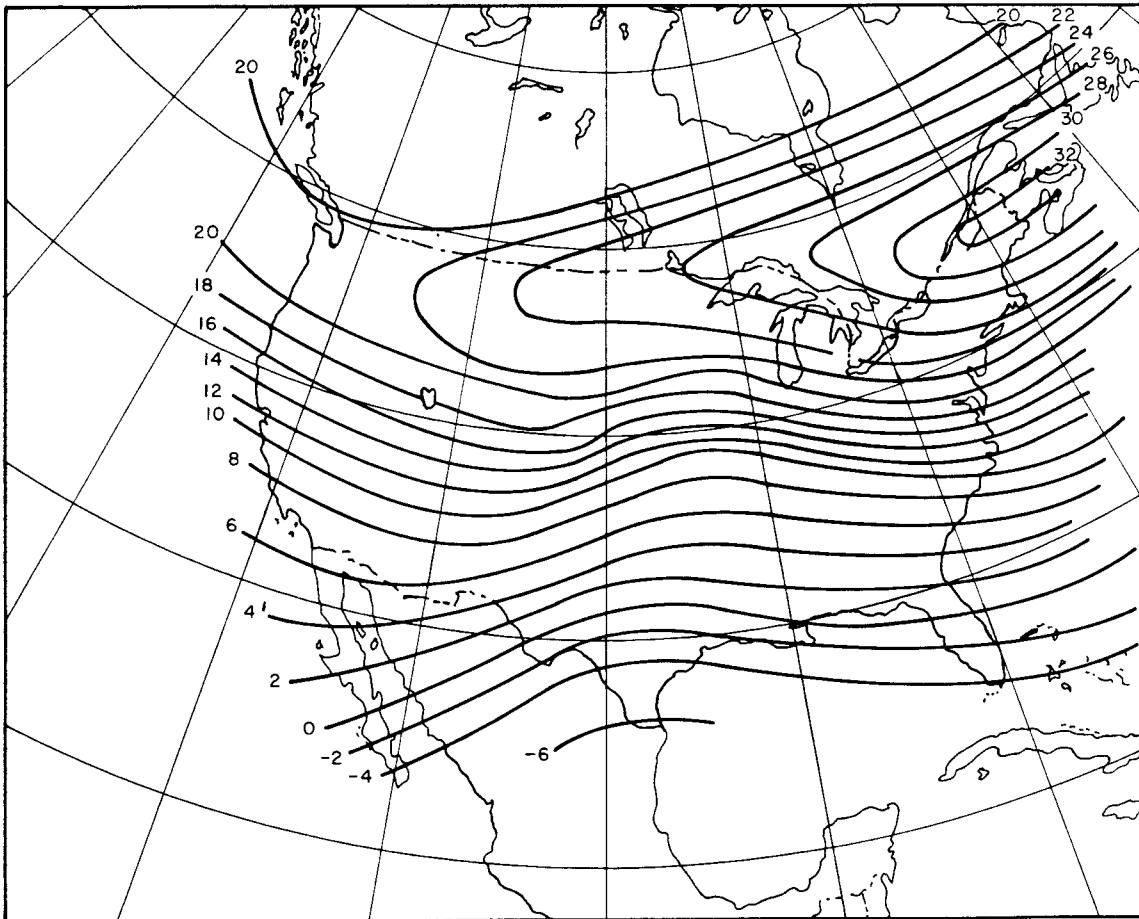


Fig. 2—The summer mean east-west component of the wind (\bar{u})

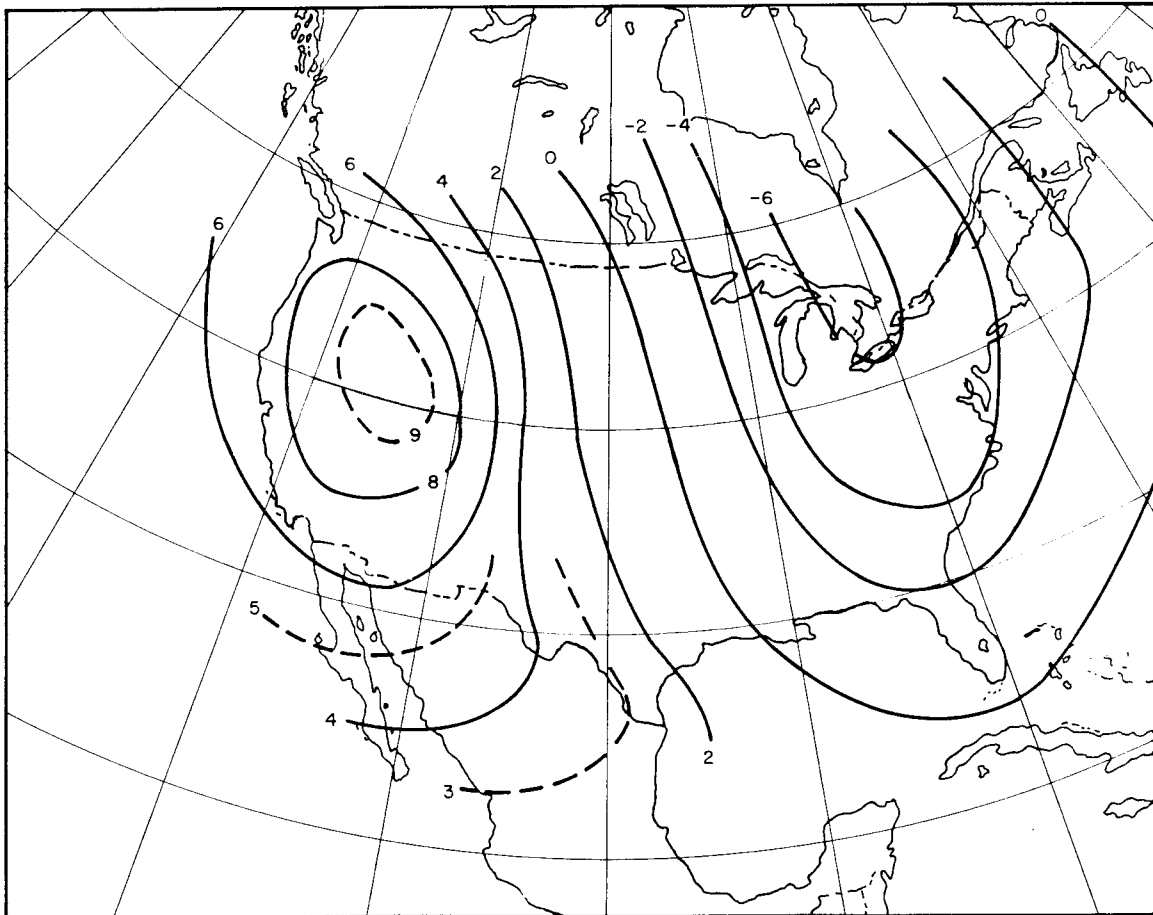


Fig. 3—The summer mean north-south component of the wind (\bar{v})

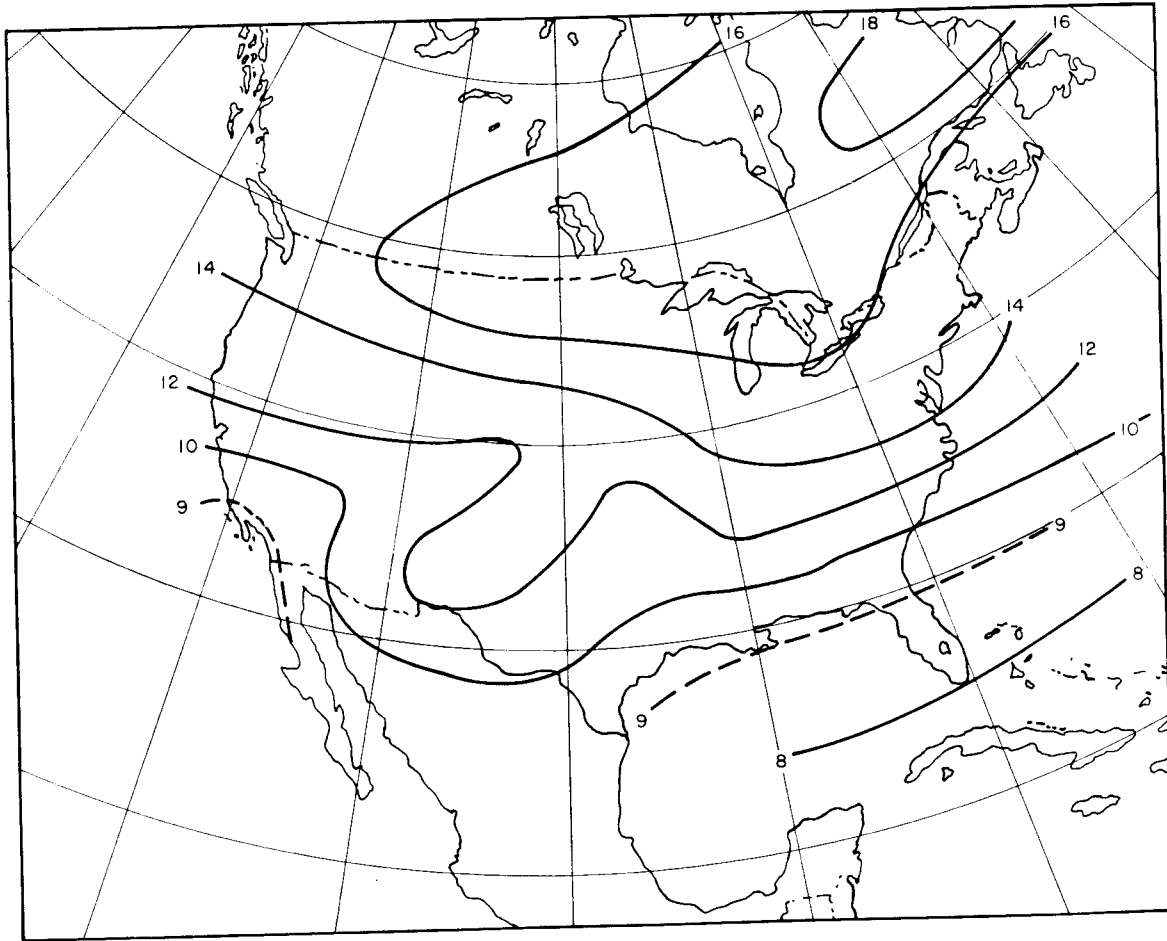


Fig. 4—The summer standard deviation of the wind components (s)

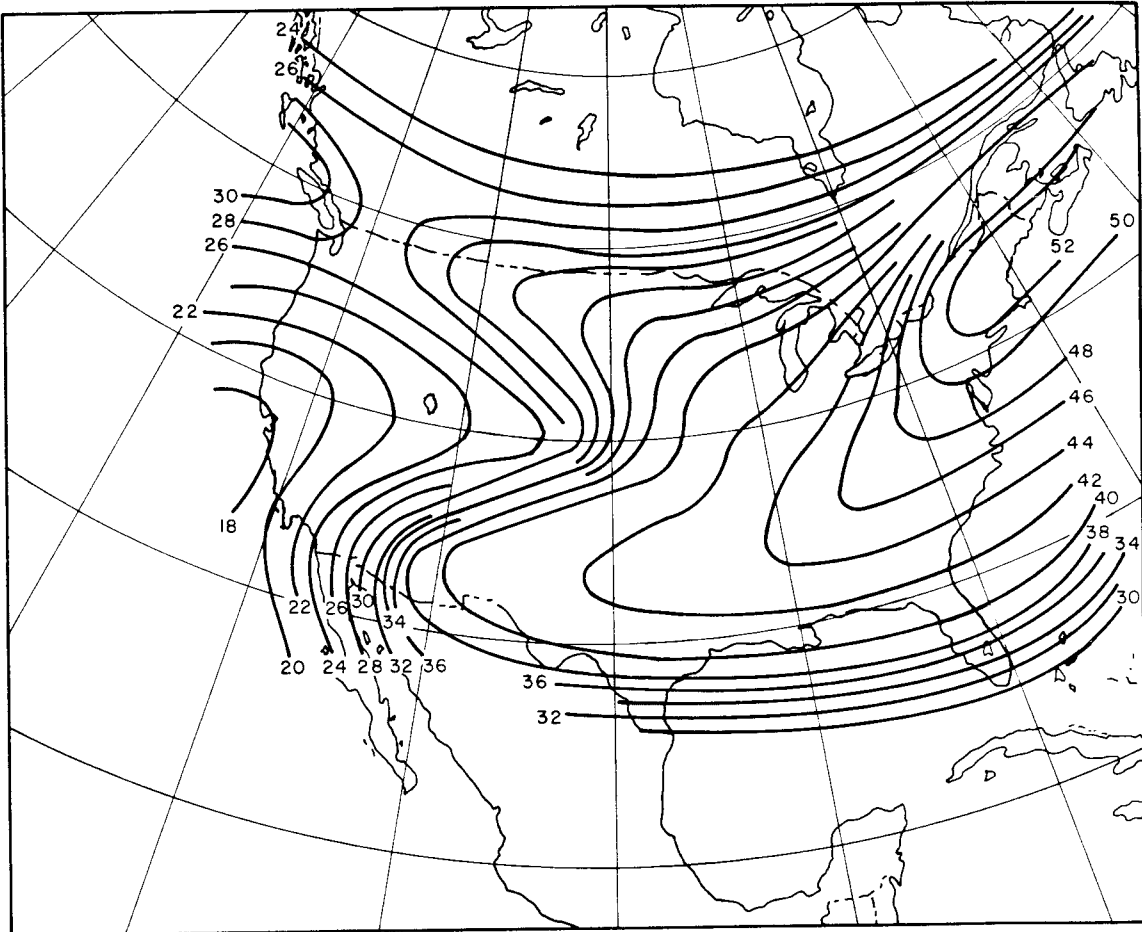


Fig. 5—The winter mean east-west component of the wind (\bar{u})

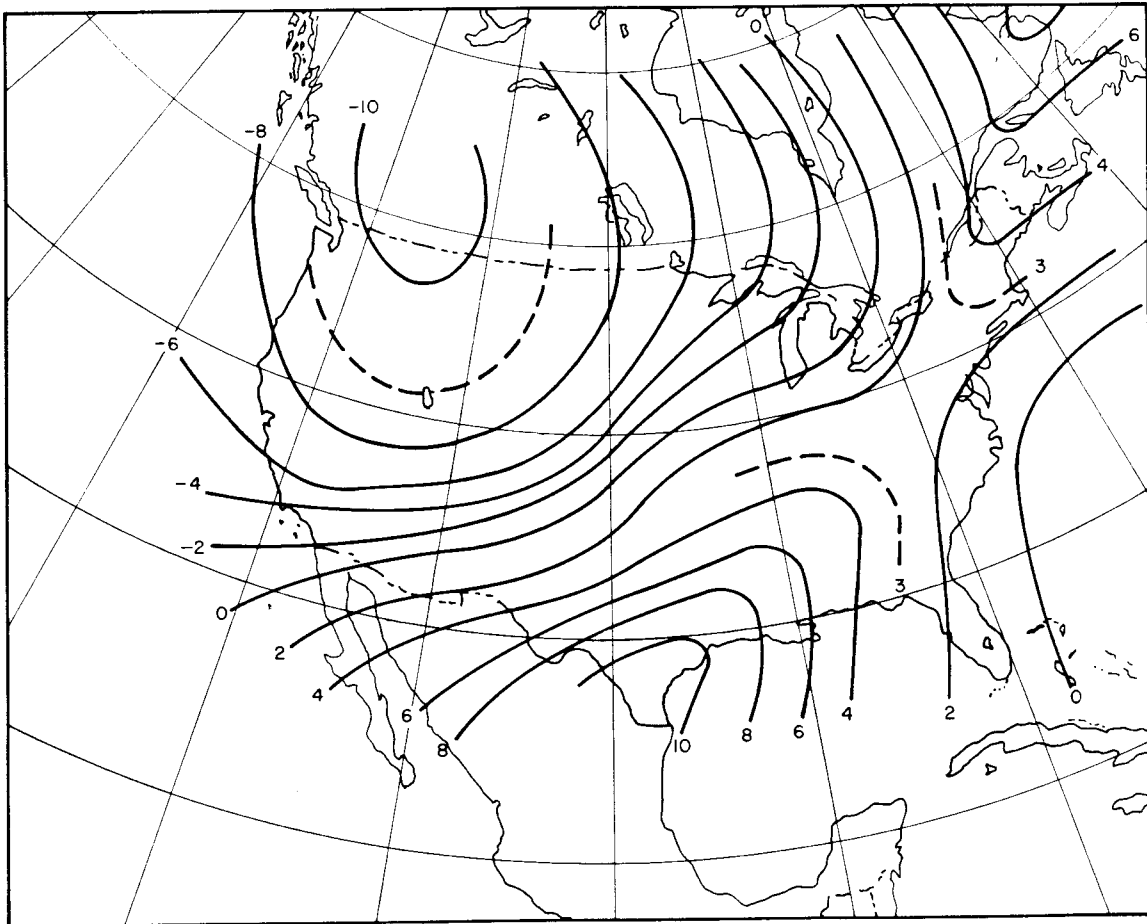


Fig. 6—The winter mean north-south component of the wind (\bar{v})

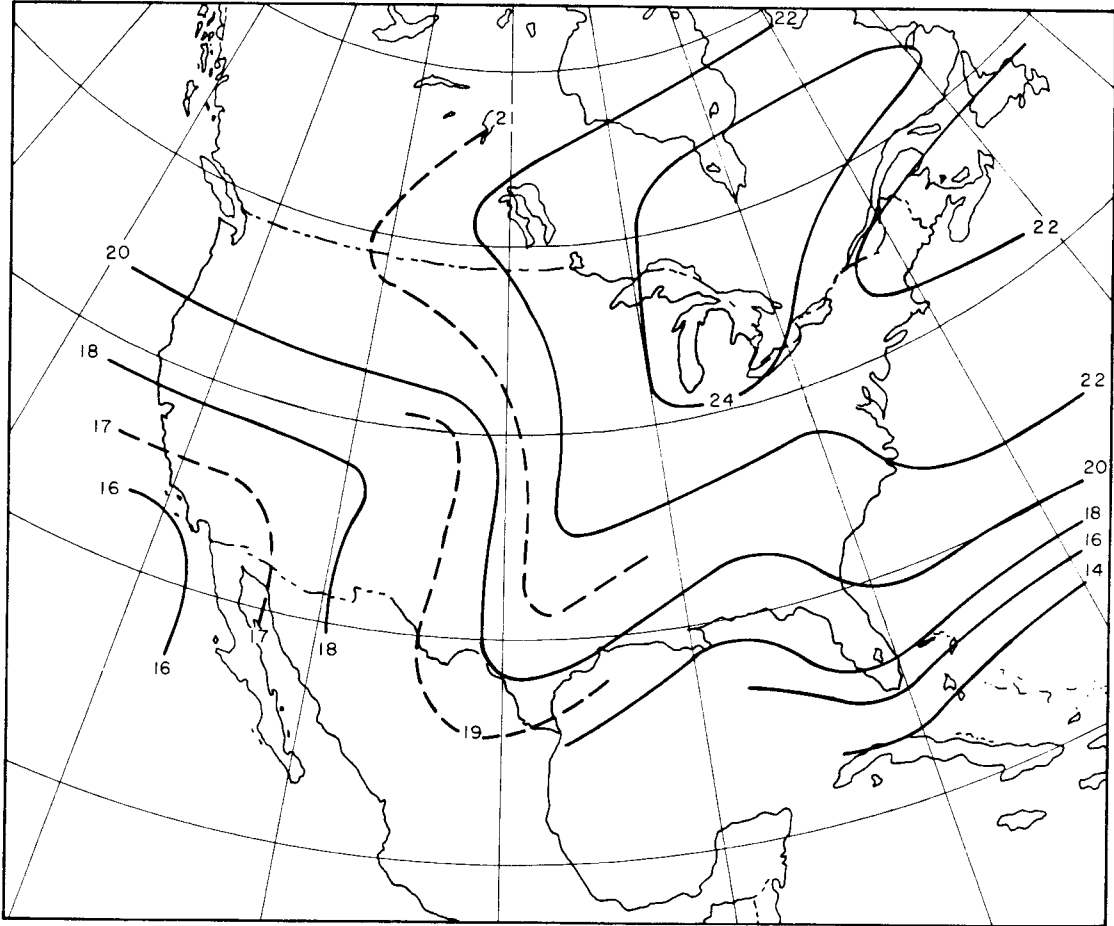


Fig. 7—The winter standard deviation of the wind components (s)

IV. PROBABILITY COMPUTATIONS

Section II showed that the dose or dose rate received at a point (X, Y) is a function of the wind velocity. One can also show that, at any (X, Y) , a range of wind velocities creates doses equal to or greater than a given value. The probability that the point will receive at least the given value, we determine from the frequency function of the wind velocities if we know their limits. But because the frequency function and the limits are complex, we cannot find a simple analytical solution. A graphical method is possible,⁽⁵⁾ but tedious and time consuming. And to compute a distribution of doses, the graphical computations must be repeated for several different doses. For most problems this is impractical.

Sampling is the common statistical procedure for determining a quantity's distribution function. Convenient as this might seem, we cannot explode bombs on randomly chosen days and then measure the dose received at various locations. Fortunately, the conditions of an attack can be simulated on a computer. The wind statistics for the area are fed to a computer that picks winds at random from the given distribution and then computes from the fallout model the radiation dose received at various locations. After many repetitions, the distribution function is formed. Such a procedure, seeking a statistical estimate of a quantity by random sampling from an artificial population that is, in some way, a model of the real physical system, is called a Monte Carlo method.

A simple flow diagram, Fig. 8, outlines the steps in the computation. We will discuss each of the steps in the paragraphs that follow.

(a) Pick random u and v components of the wind.

According to the central limit theorem, if x has a distribution with mean μ and standard deviation σ for which the moment generating function exists, then the variable $t = (\bar{x} - \mu) \sqrt{n}/\sigma$ has a distribution that approaches the standard normal distribution as n approaches infinity.⁽⁶⁾ Thus, with a computer routine capable of generating random numbers from a uniform distribution, we can compute a random number, t , that has a distribution approximating the standard normal distribution. The random components of the wind are then:

$$u = \bar{u} + St_w$$

$$v = \bar{v} + St_v$$

where u and v are random values of the east-west and north-south winds, \bar{u} and \bar{v} are the mean east-west and north-south winds, S is the standard deviation of the winds, and t_u and t_v are independently determined random numbers from a standard normal distribution. Values of \bar{u} , \bar{v} and S are found in Figs. 2-7.

(b) Compute downwind and crosswind distances.

The fallout computations are performed in a coordinate system centered at the target and oriented with the downwind axis (X) parallel to the direction of the wind. For each new target and wind combination, the coordinate axes must be translated and rotated into that orientation.

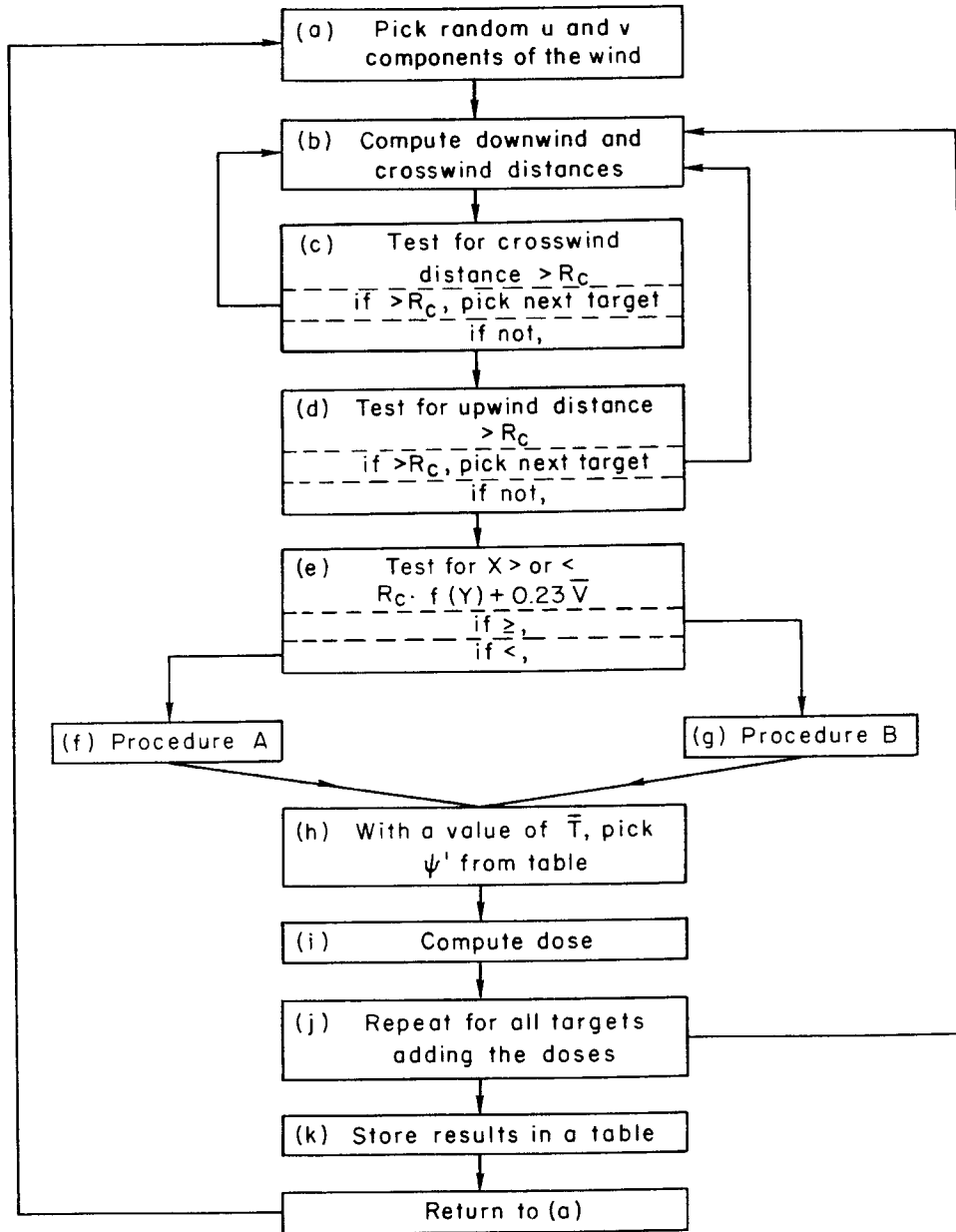


Fig. 8 — Computational procedure

If ϕ and λ are the latitude and longitude of the target, and Φ and Λ are the latitude and longitude of the point for which the fallout computations are performed, then

$$x \approx 60 (\lambda - \Lambda) \cos \Phi^*$$

and

$$y = 60 (\Phi - \phi)$$

where x is the east-west distance between the target and the point, and y is the north-south distance between the target and the point. The units are in nautical miles. With these values of x and y , the downwind distance between the point and the target, X , and the crosswind distance between the point and the target, Y , become

$$X = \frac{y v + x u}{V}$$

$$Y = \frac{y u - x v}{V}$$

where

$$V = \sqrt{u^2 + v^2}$$

(c) Test for crosswind distance greater than R_c , and

(d) Test for upwind distance greater than R_c .

These tests determine the position of the point (X, Y) relative to the boundaries of the fallout pattern. If the point lies within the

* This equation implies that the computations are made on a plane tangent to the earth at the point (X, Y) .

path of fallout from the target, the dose received is computed. If the point is further (either upwind or crosswind) than a cloud radius from the target, the target is rejected and the next target picked.

(e) Test for X greater or less than $R_c f(Y) + 0.23 \bar{V}$.

In Section II we found that the equations used for \bar{T} and ΔT depend on the position of the point relative to the initial cloud. This test determines the relative position of the point and selects the proper set of equations.

(f) Procedure A.

If the downwind distance, X, is less than $R_c f(Y) + 0.23 \bar{V}$, procedure A is selected and Eqs. (9a) and (9b) are used.

(g) Procedure B.

If the downwind distance, X, is greater than or equal to $R_c f(Y) + 0.23 \bar{V}$, procedure B is selected and Eqs. (10a) and (10b) are used.

(h) With a value of \bar{T} pick ψ' from a table.

A table of ψ' values formed from Fig. 1 is used as an input to the program. If weapon yields other than one megaton are used, it will be necessary to compute \bar{T}^* from Eq. (8). Thus, ψ' values for all yields can be found from a single table. For values of ψ' not in the table, linear interpolation may be used.

(i) Compute dose.

Equations (5), (9), and (11) are used to compute the dose at the point (X, Y).

(j) Repeat for all targets adding the doses.

The dose received from each target is computed and summed. The result is the total dose received from an attack on all the targets when the wind has the value selected in Step (a).

(k) Store results in a table.

A table can be formed by dividing the expected range of doses into class intervals. The results can be stored in this table by either counting the number of times the dose falls within each class or by counting the number of times the dose is less than the upper limit of each class. The latter procedure will provide the distribution function. The program may also be written to include computations of the mean and the standard deviation of the doses.

A program similar to the one outlined here was written for the JOHNNIAC* and applied to a typical complex of targets near a city. The results are presented in the next section.

*A RAND digital computer.

V. TYPICAL RESULTS

The target system we used in this example is shown in Fig. 9. The results, of course, depend on the number and type of weapons used, the wind statistics, and the value of the factor M in Eq. (11). The assumptions made for this attack are listed in the Table. In the final section we will discuss the effect of changing the values of these parameters.

Table
ASSUMPTIONS

Yield of weapons	5 MT
Number of weapons	one per target
Fission fraction	0.5
M	$15 \times 10^5 \text{ r/hr/MT}/(\text{n mi})^2$
Summer winds	
\bar{u}	7.0 kn
\bar{v}	6.4 kn
S	11.9 kn
Winter winds	
\bar{u}	39.6 kn
\bar{v}	1.1 kn
S	17.8 kn

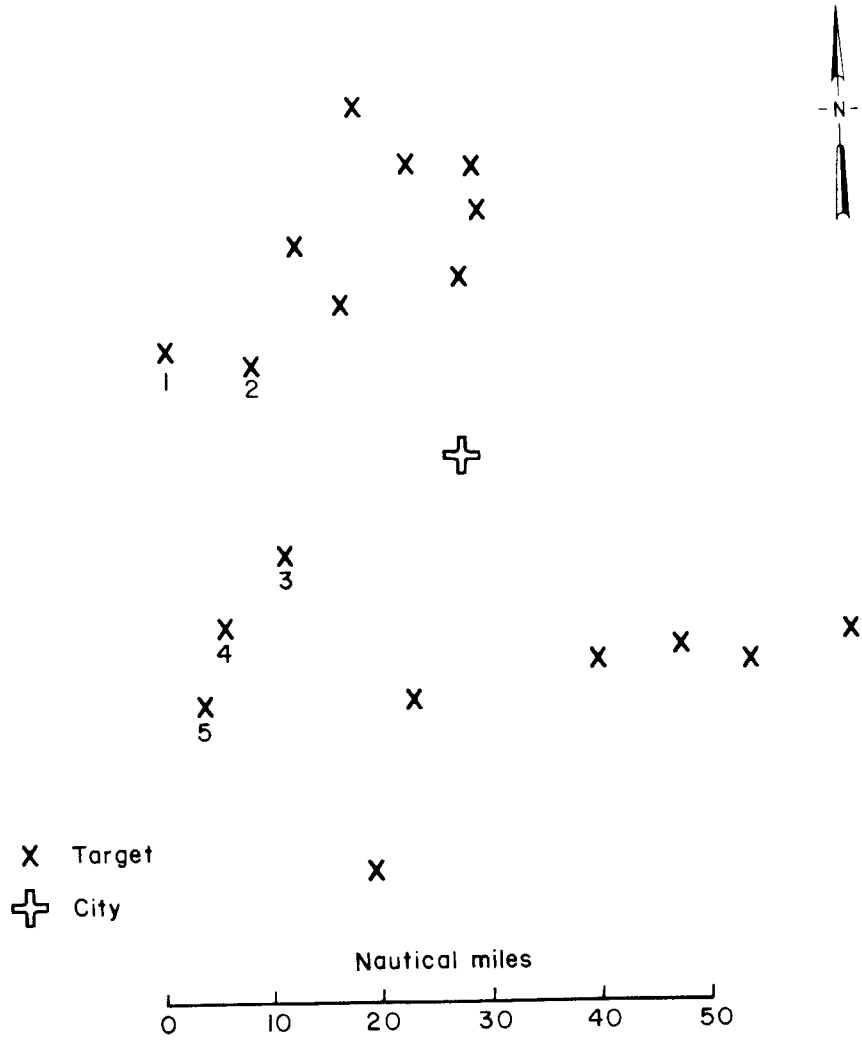


Fig. 9—Location of targets relative to the city

Figures 10 and 11 plot the distribution of the 48-hour integrated dose received at the city during the winter and summer seasons. These figures show the probability of receiving a given dose or greater. For example, during winter, the probability of receiving 3000 or more roentgens in a 48-hour period is about 40 per cent, while the probability of receiving 1000 or more roentgens is close to 90 per cent.

The slope of the winter curve indicates that the probability of receiving small doses at the city rapidly approaches 100 per cent as the dose approaches 400 r. Thus, the city will always receive at least 400 r. The extent of the area covered by this probability is important. By repeating the computations for a grid of points in the area around the targets, the horizontal distribution of the probability can be determined. For example, Fig. 12 shows the winter horizontal distribution of the probability of receiving at least 100 r. This region extends about 50 nautical miles to the east and about 20 nautical miles to the north and south of the city. The summer situation is different. According to Fig. 11, the city will escape fallout 3 per cent of the time. An explanation for this will be given in the final section.

The average dose received at each grid point was computed for a sample of fifty winds. The results are shown in Fig. 13.

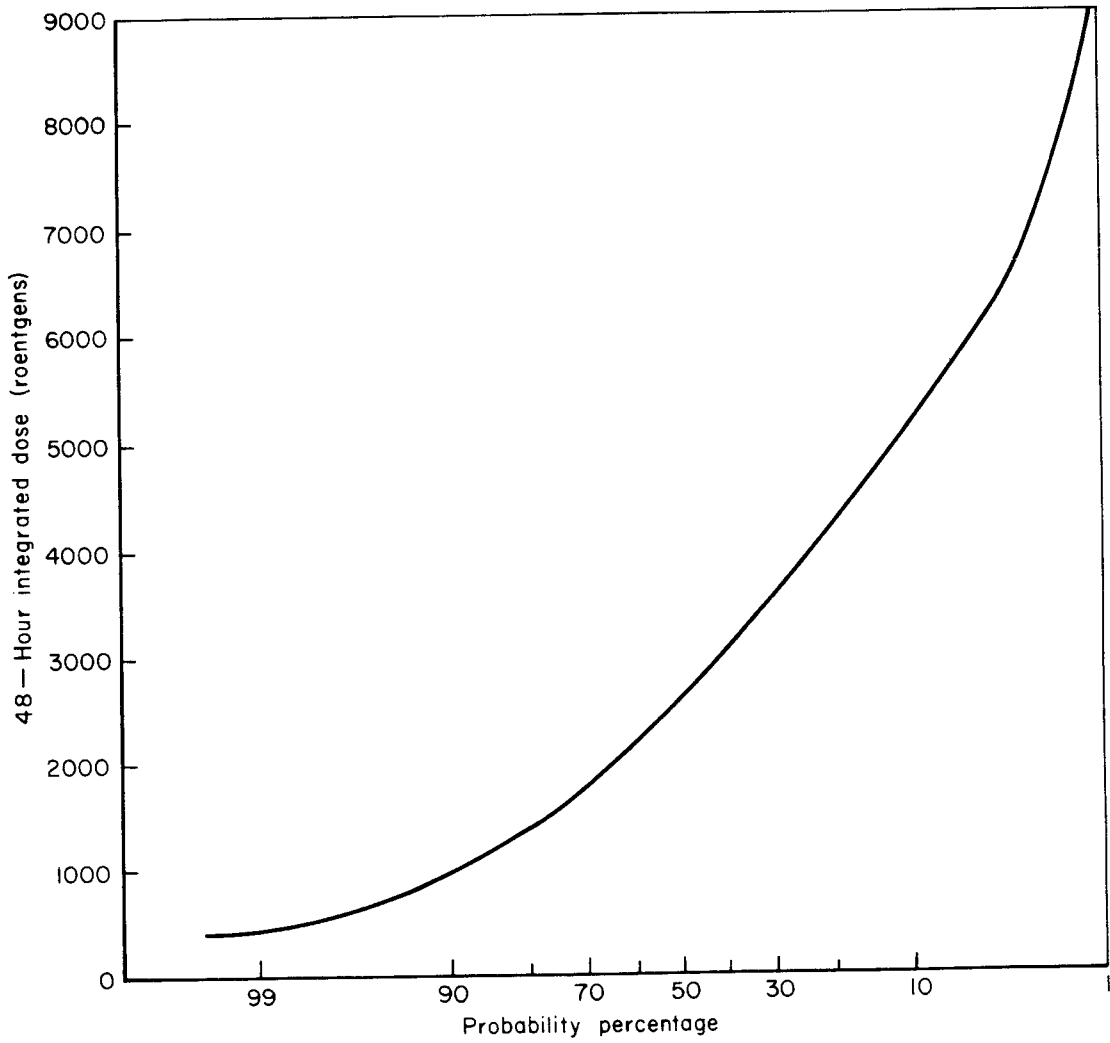


Fig. 10 — Probability of receiving greater than a given dose (winter season)

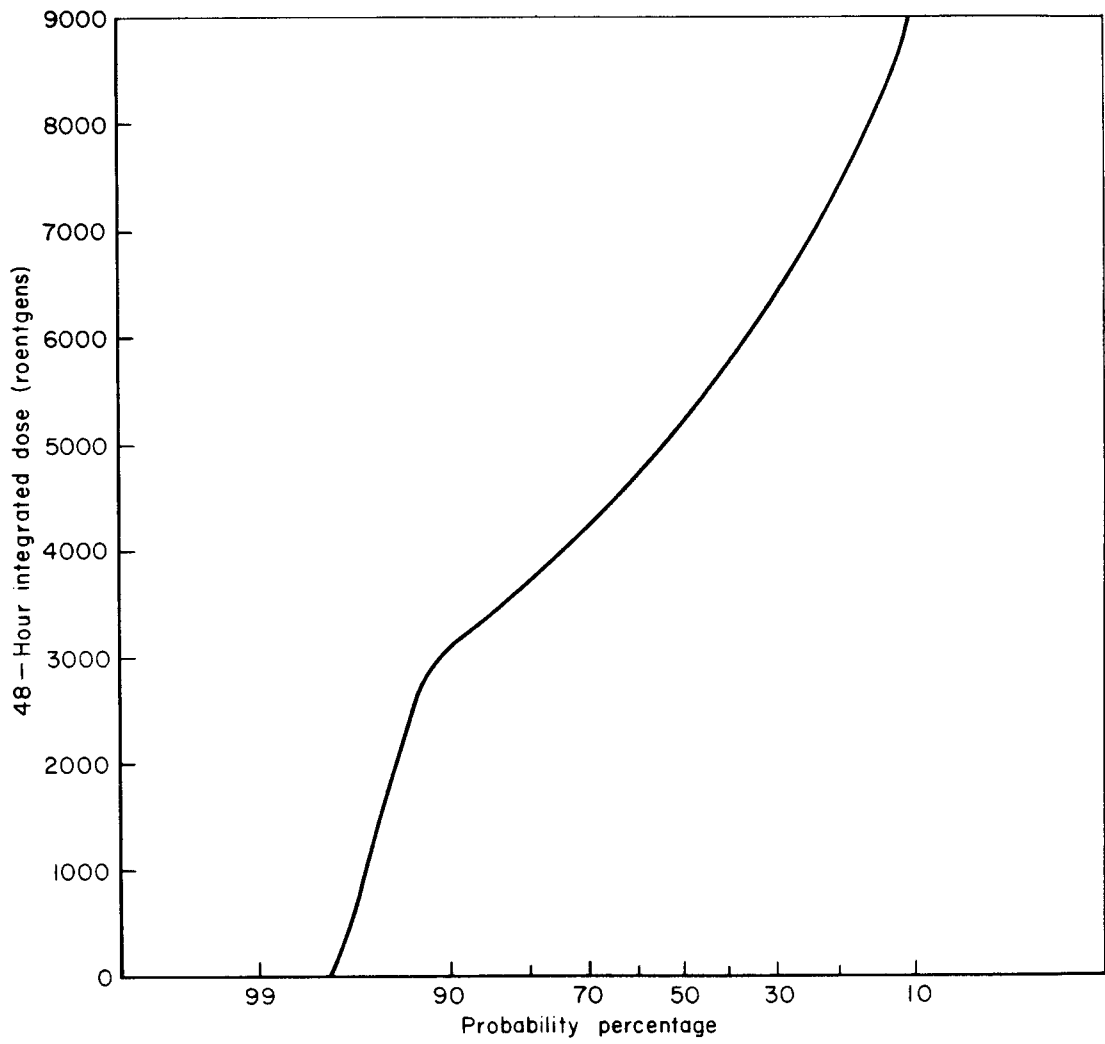


Fig. II — Probability of receiving greater than a given dose (summer season)

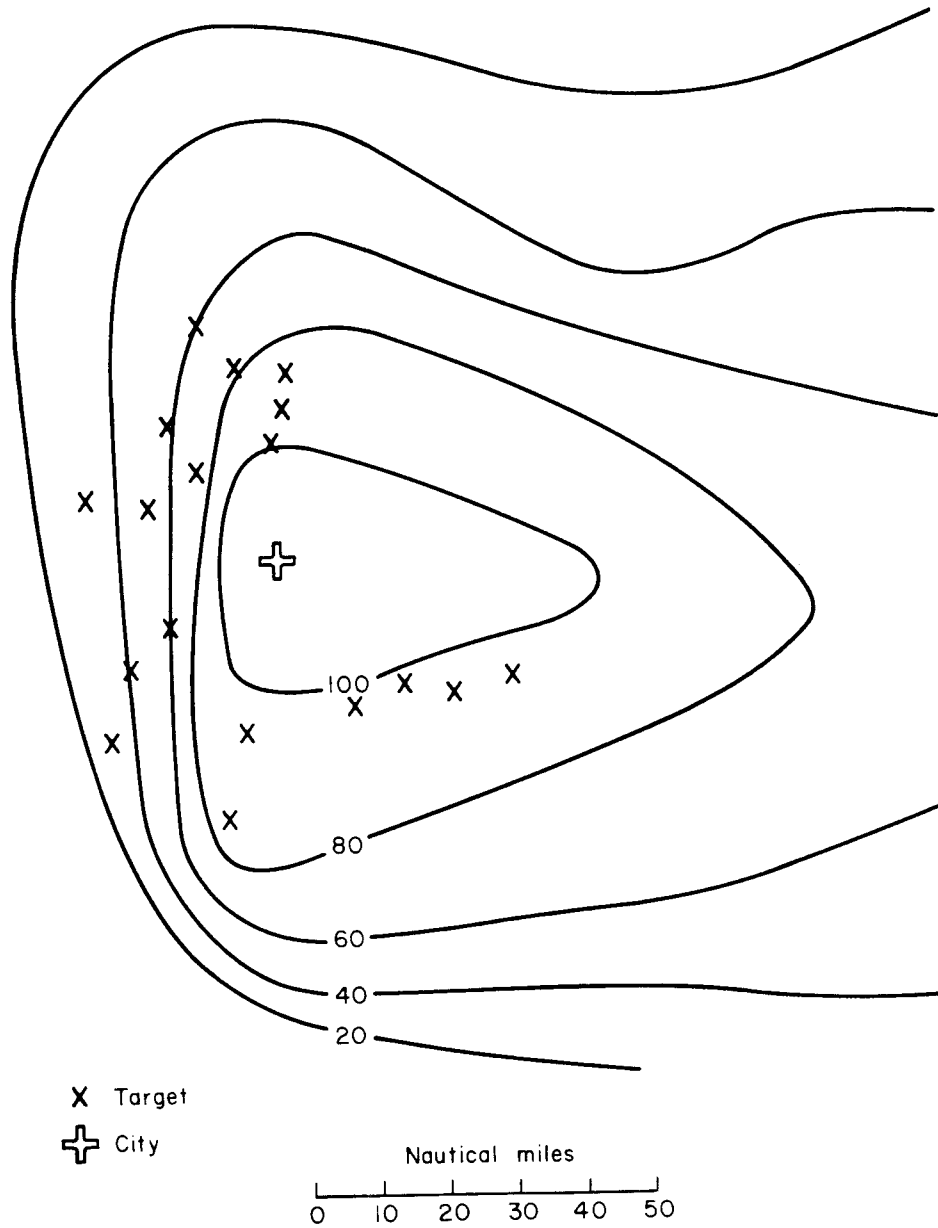


Fig. 12—The horizontal distribution of the probability of receiving greater than a 100-roentgen 48-hour integrated dose (winter season)

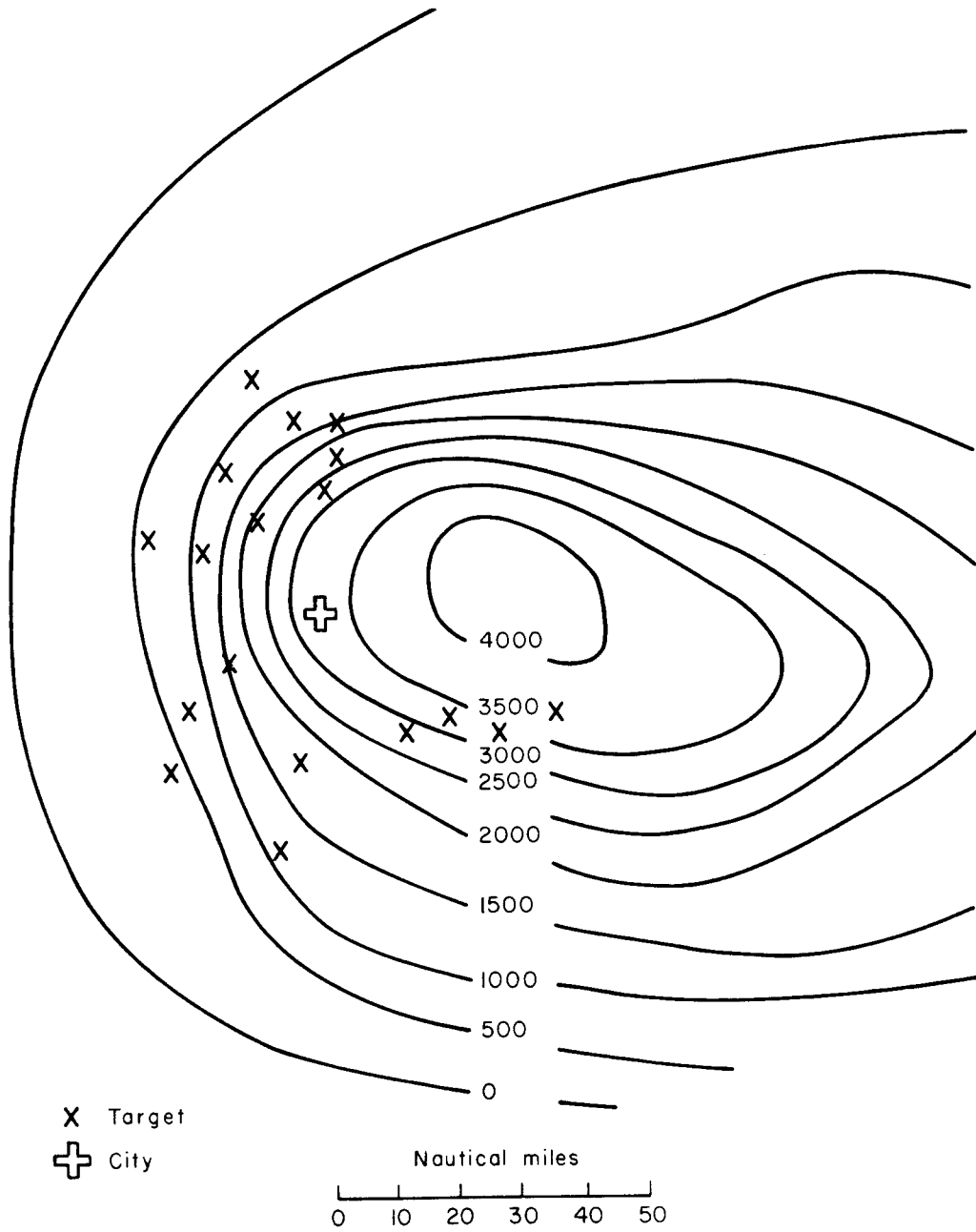


Fig.13—Horizontal distribution of expected 48-hour integrated dose (winter season)

VI. CONCLUSIONS

We began by saying that uncertainties in the original information would affect the results of the prediction models. In this Memorandum we have outlined a procedure to eliminate one of the uncertainties — the wind. This procedure allows us to evaluate fallout effects for all wind conditions. There are, of course, other uncertainties to consider. Among them are the yield of the weapons, the fission fraction of the weapons, the dose rate at one hour per megaton of fission per square nautical mile, and the target system. It should be appropriate at this point to demonstrate how uncertainties in these parameters will affect the results. But first, a final comment on the wind is warranted.

In the fallout model we have approximated the wind affecting the fallout by a vertically averaged wind vector, and we have replaced the parameters of a single wind (u and v) by the parameters of the wind statistics (\bar{u} , \bar{v} , and σ). Finally we computed at a point, not the dose, but a distribution function. Needing clarification is how the assumption of a vertically averaged wind and how differences in the standard deviations and means of the wind affect the distribution function.

In Section III we suggested that the error introduced by neglecting directional shear would be small. A quantitative discussion of this error would be difficult without first computing the distribution function with a fallout model that includes shear. However, the nature of the error can be deduced from some simple arguments. The distribution

function should be least affected at points near ground zero (within two or so cloud radii) where, for all but the lightest winds, there would be insufficient time for appreciable lateral spreading. Farther from ground zero, the fallout pattern might be much wider than a cloud diameter. Here, with shear, we would expect fewer large doses and more small doses. Thus the distribution function would so change that lower doses would occur with a higher probability and higher doses with a lower probability. We expect the change to be much smaller in magnitude than those in the distribution function produced by variations in the other parameters.

Differences in the distribution function produced by the wind statistics are closely related to target position. An excellent example of this are the summer and winter distribution functions presented in the last section. The curves of Figs. 10 and 11 are compared in Fig. 14. The probabilities are higher in summer for doses greater than 600 r. The reverse is true for doses less than 600 r. These features can be explained by differences between the summer and winter-wind statistics and the positions of the targets relative to the city.

The value and position of the maximum dose are sensitive to the wind velocity. Its value is lower and its position is farther from ground zero with strong winds than with light winds. From the statistics given in the Table, we see that winter winds usually range between 22 and 58 kn, while summer winds are usually less than 22 kn. These lighter summer winds cause the generally higher radiation levels then observed at the city. We can see from Fig. 13 that the stronger winter winds will produce the highest levels outside the city. If the city

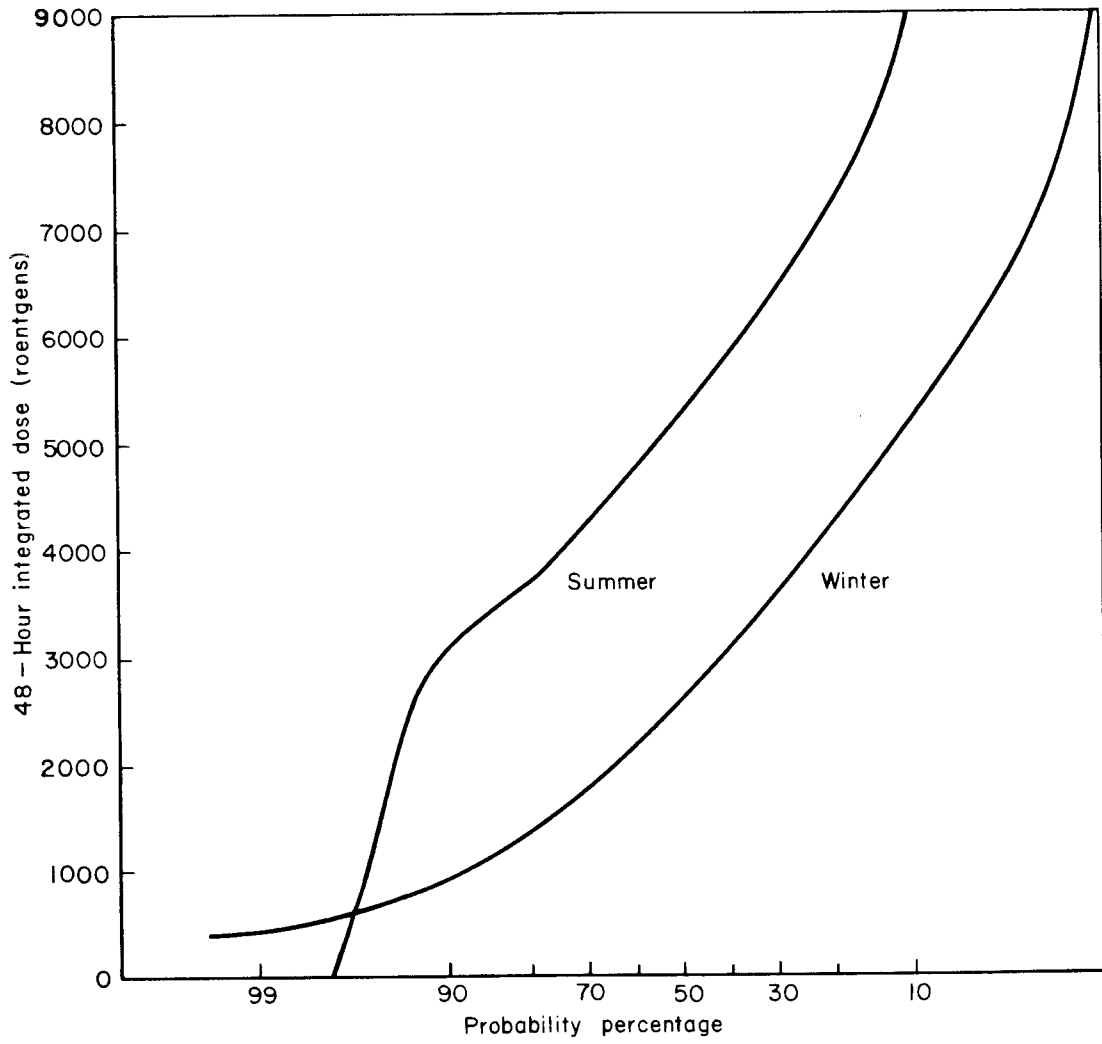


Fig. 14 — Comparison of the summer and winter distribution functions

were farther east, though, the winter distribution function might indicate higher doses and the summer distribution function lower doses.

To explain the behavior of the summer distribution functions below 3000 r, we must look for differences between the summer and winter standard deviations. In winter, the standard deviation is less than half the magnitude of the mean wind; in summer, it is slightly larger than the mean wind. As a consequence, the winter wind direction will almost always be within 90 degrees of the mean, while in summer there will be a significant probability of the wind blowing from all directions. In the winter, since the mean wind is from the west, the winds will bring fallout to the city from one or more of the several targets located to the west. During the summer, however, the wind may be from the east or northeast, where there are no targets. Thus, there is a small probability of no fallout reaching the city in the summer months. Figure 14 shows this probability to be about 3 per cent. Of course if there were targets in the eastern sector, the light summer winds would produce high radiation levels at the city regardless of the wind direction. The summer distribution function would then approach a constant dose value as the probability approaches 100 per cent.

So much for the wind. One of the other parameters determining the amount of radiation available for fallout is the yield. If we change it, while holding the other parameters constant, we can produce large variations in the distribution function. For example, Fig. 15 shows the wind distribution functions for yields of one, five, and ten megatons. With ten megatons, the probability of receiving doses

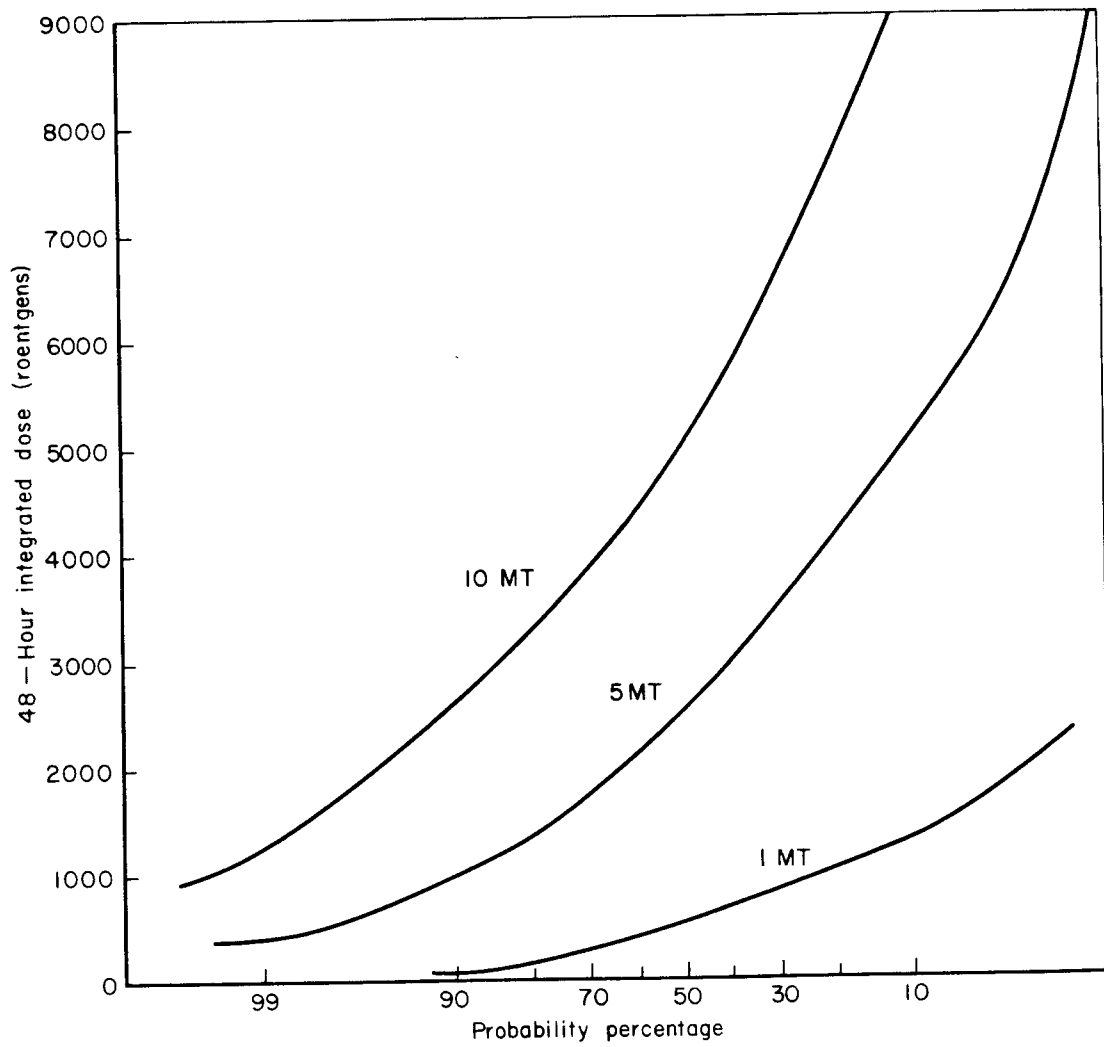


Fig.15—Winter distribution functions for three yields

greater than 1000 r is 99.5 per cent; with one megaton, it is 20 per cent. For doses greater than 2000 r, the range is from 95 to 3 per cent.

The other parameter controlling the amount of radiation produced is the product Mf (Fig. 16). We computed the winter distribution function for three values of Mf ; 15×10^5 , 7.5×10^5 , and 3.75×10^5 . These values of Mf were obtained from reasonable estimates of M and f . For the first, M equalled 20×10^5 r/hr/MT/(n mi)² and f equalled 0.75; for the last, M equalled 13×10^5 r/hr/MT/(n mi)² and f equalled 0.25. For the first estimate, the probability of receiving a dose of 2000 r or more is 89 per cent; for the last, it is 24 per cent.

By removing the targets numbered 1 through 5 in Fig. 9, we investigated the effect of targeting on the results. These targets were deliberately picked to produce the largest change in the winter distribution function. Fig. 17 compares the results for the original target system with those for the new target system. Some of the difference between curves is a result of deploying a small number of weapons. Most, however, is a result of removing the five targets to the west of the city. When this is done, there is a significant decrease in the city's radiation level. The probability of receiving 1000 or more roentgens drops from 90 per cent to 28 per cent.

Although we now have a way of estimating the effect of wind variability on the fallout, the results are highly sensitive to several unpredictable parameters. Indeed, changing the assumptions of weapon characteristics and targeting can produce variations in the results that are as large as, or greater than, the seasonal variations

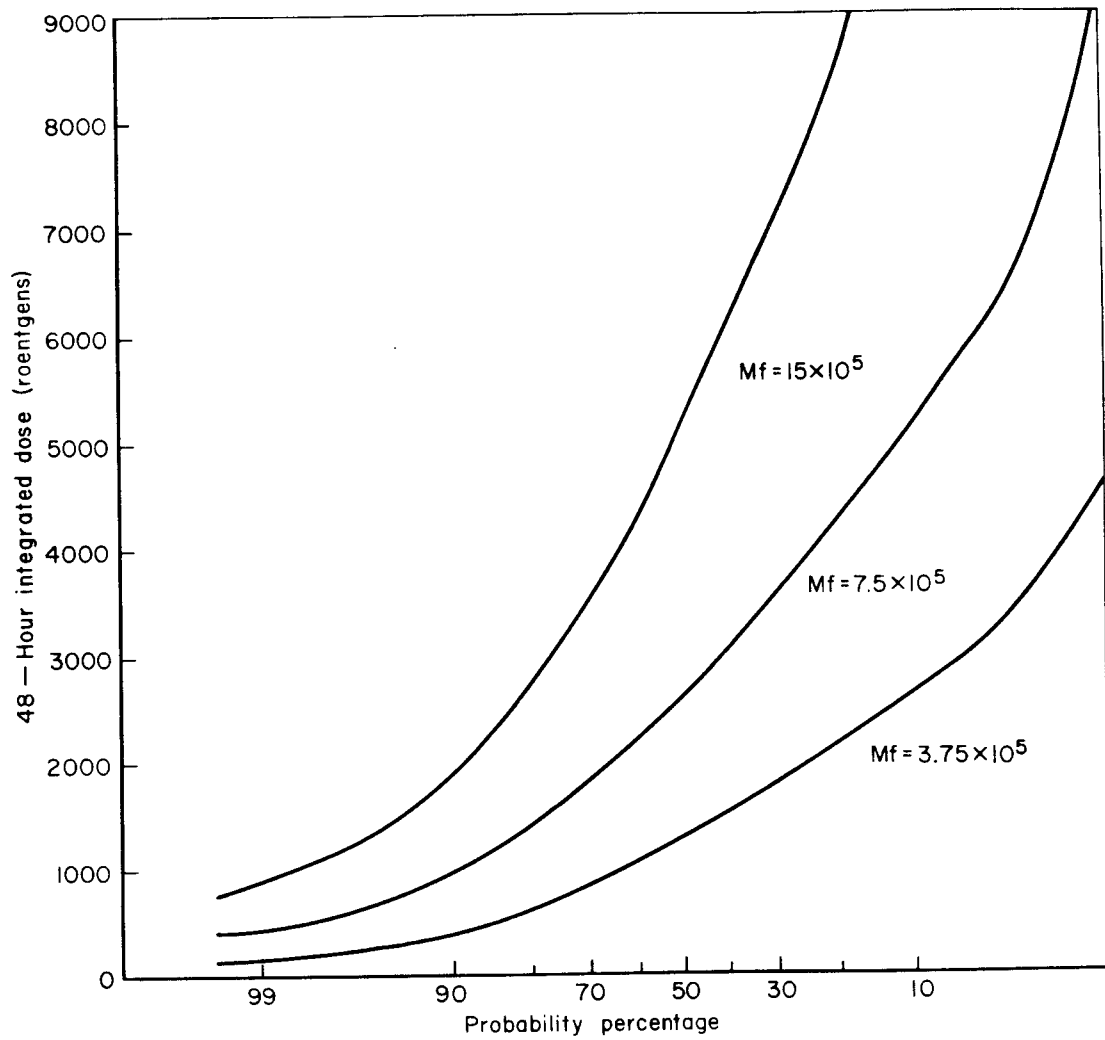


Fig. 16 — Winter distribution functions for three values of Mf

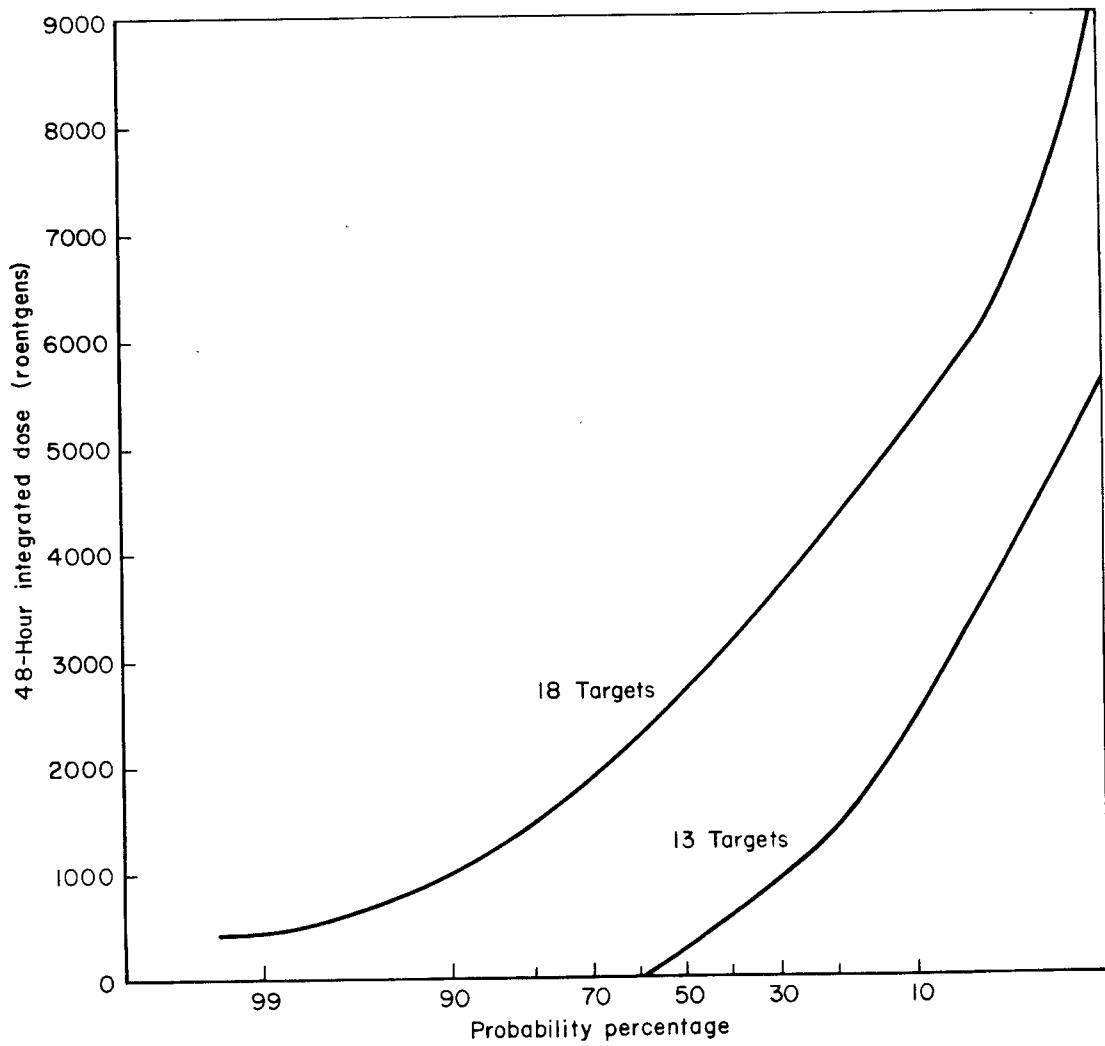


Fig. 17—Winter distribution functions for two target systems

of the distribution function. A prudent choice of all parameters is necessary. Even then, the results must be evaluated in the light of other weapons and targets.

REFERENCES

1. Greenfield, S. M., Radioactive Contamination from a Multibomb Campaign, The RAND Corporation, RM-1969, January 1956.
2. Kellogg, W. W., R. R. Rapp, and S. M. Greenfield, "Close-In Fallout," J. Met., Vol. 14, No. 1, February, 1957, pp. 1-8.
3. Batten, E. S., D. L. Iglehart, and R. R. Rapp, Derivation of Two Simple Methods for the Computing of Radioactive Fallout, The RAND Corporation, RM-2460, February 18, 1960.
4. Kellogg, W. W., Atmospheric Transport and Close-In Fallout of Radioactive Debris from Atomic Explosions, The RAND Corporation, P-109, May 20, 1957, revised August 26, 1957.
5. Batten, E. S., A Method of Computing Fallout Hazard For Areas Near a Nuclear Blast, The RAND Corporation, RM-2734, June 27, 1961.
6. Hoel, P. G., Introduction to Mathematical Statistics, John Wiley and Sons, Inc., New York, 1956.